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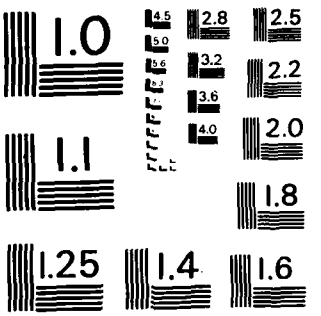

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STUDY OF FINITISTIC CHANNEL MODELS

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The modeling of discrete-time stationary communication channels with memory was investigated. The approach was to develop distance measures to quantify the degree to which one channel or model approximates another and then to characterize the class of channels that can be well approximated by various types of models. A variety of channel distances were investigated. The results provide characterizations of the classes of channels approximable by finite memory models, primitive models, and finite state indecomposable models. The concept of channel entropy, which is the minimum amount of randomness needed to

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Abstract

The modeling of discrete-time stationary communication channels with memory was investigated. The approach was to develop distance measures to quantify the degree to which one channel or model approximates another and then to characterize the class of channels that can be well approximated by various types of models. A variety of channel distances were investigated. The results provide characterizations of the classes of channels approximable by finite memory models, primitive models, and finite state indecomposable models The concept of channel entropy, which is the minimum amount of randomness needed to simulate a channel, was discovered. The results apply to both discrete and continuous alphabet channels.

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1. Research Objectives and Results

Introduction

This report describes the research performed under AFOSR Grant Number 80-0054, which extended from January 1, 1980 to December 31, 1982. This research addressed the problem of modeling discrete-time communication channels with memory. The objectives were to investigate a variety of finitistic models and to determine what channels they could accurately model. The approach taken was to develop appropriate channel distance measures (i.e., quantitative measures of the degree to which one channel or model approximates another) and then to characterize the class of channels that can be approximated with arbitrary accuracy (according to the appropriate distance measure) by finitistic channel models of a given type, e.g. finite memory, finite state or primitive.

The research was divided among several areas: channel distance measures, channel approximation and representation, and extensions to continuous alphabets. The results in these areas are described in the subsections below, following a brief summary of the preliminary work upon which the research is based. Unless otherwise stated all channels are assumed to be stationary and to have finite alphabets. The references listed in brackets refer to the list of publications given in Section 2.

Summary of preliminary work

In preliminary work,[1] the $\bar{d}$-channel distance was introduced as a measure of the degree to which one finite alphabet stationary channel or model approximates another. This distance is a natural extension of Ornstein's $\bar{d}$-distance for

[1]D. L. Neuhoff and P. C. Shields, "Channels with almost finite memory," *IEEE Trans. Inform. Theory*, vol. IT-25, pp. 440-447, July 1979.

random processes.[2] Then the class of channels that are approximable with arbitrary accuracy by finite memory models was identified. Such channels turned out to be characterized by the properties of $\bar{d}$-continuity and conditional almost block independence (CABI). The former is an input memory decay property introduced by Gray and Ornstein[3] and the latter is an output memory decay property. In addition it was shown the class of channels approximable by finite memory models (namely, the $\bar{d}$-continuous CABI channels) is identical to the class of channels approximable by elementary finite memory models called *primitive* in which the output is simply a finite sliding-block coding of the input joined with a stationary memoryless (I.I.D.) source of noise. Primitive models, therefore, play a fundamental role.

Channel distance measures

Ideally, the distance measure one uses in a theory of channel approximation should be a metric or pseudometric and should be strong enough that channels adjudged to be close have similar behavior and weak enough that channels without significant differences are adjudged to be close. More specifically one would like to use the weakest distance having the property that if two channels are close in this distance, then their channel capacities are close and the respective performances obtained using a fixed channel code are also close. In other words capacity and code performance should be continuous functions of the channel with respect to the chosen channel distance measure. The $\bar{d}$-distance is a pseudometric and is strong enough to have the desired capacity and code performance continuity. Furthermore, if two channels have $\bar{d}$-distance 0, then they are *equivalent* in the sense that for any stationary or

[2]D. S. Ornstein, "An application of ergodic theory to probability theory," *Ann. Prob.*, vol. 1, pp. 43-65, 1973.

[3]R. M. Gray and D. S. Ornstein, "Block coding for discrete stationary $\bar{d}$-continuous noisy channels," *IEEE Trans. Inform. Theory*, vol. IT-25, pp. 292-306, May 1979.

block-stationary input process applied to each, the respective input/output pair processes are identical. From the communications point of view, equivalent channels are indistinguishable. On the other hand the $\bar{d}$-distance is so strong that it assigns nonzero distance to some pairs of equivalent channels. Hence it would seem prudent to look for weaker distances.

In this research four weaker distance measures were studied, which we here denote d_1, d_2, d_3, d_4 in decreasing order of strength. Each is a pseudometric, all have the desired capacity continuity, all but d_4 (the weakest) have the desired code performance continuity and d_2, d_3 and d_4 assign zero distance to equivalent channels. Thus it would appear that d_3 is the most appropriate distance to use in a channel approximation theory. However, it was also discovered that for the important class of $\bar{d}$-continuous channels $\bar{d} = d_1 = d_2 = d_3$ and $\bar{d}$ is uniformly equivalent to d_4. Thus for the most important case the original $\bar{d}$-distance has all the desired properties. This is fortunate because it is also the easiest distance to work with. This work is described in [1].

Channel approximation and representation

Channel entropy and primitive approximation

In the preliminary work, the primitive models that were demonstrated to approximate the $\bar{d}$-continuous, CABI channels employed memoryless noise sources with continuous alphabets. From a practical standpoint it is more desirable to have primitive models with finite alphabet noise. Under this research grant we were able to show that the channels approximable by primitive channels with infinite alphabet noise are also approximable by primitive models with finite alphabet noise. Thus there is no loss in restricting attention to primitive channels with finite alphabet noise. This raises the question of just how small the noise alphabet can be. This is of interest because the cost of simulating a

channel with a primitive model will in some way depend on the size of the noise alphabet. A better question is how much entropy must the noise source have. Entropy can be thought of as a measure of the uncertainty or randomness which the noise source possesses. It was discovered that to every channel there is a quantity H_C, dubbed the *channel entropy*, such that a given noise source with entropy H_N can be used to approximate (with arbitrary accuracy) any $\bar{d}$-continuous, CABI channel whose entropy H_C is less than H_N. Conversely, the given noise source cannot be used to approximate (with arbitrary accuracy) any channel whose entropy H_C is larger than H_N. Roughly speaking, channel entropy is the maximum conditional entropy of the channel output given the input, where the maximum is over all stationary input processes. (It is in some sense a dual of channel capacity.) It follows that the smallest noise alphabet with which to approximate a $\bar{d}$-continuous, CABI channel with a primitive model is the smallest integer no smaller than 2^{H_C}. This work is described in [2].

Nonanticipating channels

All channels of practical interest are nonanticipating (or equivalently causal) in the usual sense. The preliminary results on channel approximation showed that any $\bar{d}$-continuous, CABI channel, whether nonanticipating or not, could be arbitrarily well approximated by primitive channels, which could be nonanticipating or not. In other words the nonanticipating nature of real channels was not taken into account. Under this research grant it was shown that nonanticipating $\bar{d}$-continuous, CABI channels can be arbitrarily-well approximated by nonanticipating primitive channels. This work is described in [3].

Approximation by finite state models

Finite state models are the most important and widely used class of channel models. This is because they require only a finite number of parameters to

specify and because it is widely believed that they can be used to approximate a wide variety of actual channels. In this grant the latter belief was investigated by studying the class of channels approximable by finite state models.

The results obtained apply to indecomposable finite state models, which are the important subclass of finite state models in which the effect of the initial state decays with time. For example, a primitive model with a finite alphabet noise source is finite state and indecomposable. It was found that the class of channels approximable by indecomposable finite state models coincides with the class approximable by nonanticipating primitive models. This includes the class of nonanticipating $\bar{d}$-continuous, CABI channels plus, surprisingly, some anticipating channels. Again this emphasizes the importance of primitive models. As a partial converse, it was also shown that if a finite state model is $\bar{d}$-continuous and CABI and if its state equals its output, then it is indecomposable. It would also be of interest to find the class of channels approximable by arbitrary finite state models. Conceivably, this might be a larger class. However, in the course of this investigation we encountered a deep unanswered question concerning Markov chains which needs to be answered before the channel can be answered. This work is described in [3].

Approximation with respect to the weaker channel distances

If any of the distance measures weaker than $\bar{d}$ are used in a channel approximation theory, there is the potential that more channels will be approximable by primitive, finite memory, or indecomposable finite state models. Although this turns out to be so, it was found that there are not many such channels. In particular the class of channels approximable by primitive models with respect to d_i, $i=1,\ldots,4$, equals the class of $\bar{d}$-continuous, CABI channels plus all channels at d_i distance zero from this class. For d_2 this class is strictly larger, for d_3 this class is larger still, and for d_4 and d_5 this class is

the same as for d_3 and consists of all channels that are $\bar{d}$-continuous and CABI or equivalent to such. This work is described in [1].

The exact representation of $\bar{d}$– continuous CABI channels

As previously described, and $\bar{d}$-continuous, CABI channel can be arbitrarily well *approximated* by a primitive channel, which consists of a memoryless noise source and a *finite* sliding-block code that operates simultaneously on the channel input and noise source output. Under this grant it was also shown that any such channel can be *exactly* represented by a model with a memoryless noise source and *infinite* sliding-block code.

Continuous alphabet channels

In order to extend the channel approximation theory to stationary channels with continuous alphabets, a new channel distance measure, the $\bar{\rho}$-distance was introduced. The $\bar{\rho}$-distance, which is a natural extension of the $\bar{d}$ channel distance and the $\bar{\rho}$ random process distance,[4] measures the degree to which one channel or model approximates another relative to a distortion measure on the channel outputs. If the distortion measure is chosen to be Hamming distortion, then the $\bar{\rho}$-distance reduces to the $\bar{d}$-distance.

With respect to this new channel distance one may now seek to characterize the class of channels approximable by various types of models such as finite memory, primitive or finite state. Under this grant it was found that the class of channels approximable by finite memory or primitive models is characterized by the properties of weak $\bar{\rho}$-continuity and ρ-CABI, where weak $\bar{\rho}$-continuity is an input memory decay condition that generalizes and slightly weakens the property of $\bar{d}$-continuity and where ρ-CABI is the natural generalization of the CABI property to the case of continuous alphabets and distortion measure ρ. This

[4]R. M. Gray, D. L. Neuhoff and P. C. Shields, "A generalization of Ornstein's distance with applications to information theory," *Ann. Prob.*, vol. III, pp. 315-328, April 1975.

result shows that primitive channels again play a fundamental role. In the derivation of this result it was found necessary to restrict attention to channels and models satisfying an unobtrusive finite "conditional variance" condition, dubbed *wandering ρ-integrable* . This condition is satisfied by virtually all channels of practical interest. A number of results concerning the $\bar{\rho}$-topology of the class of stationary channels were also discovered. This work is reported in [4].

2. Publications resulting from the grant

[1] D. L. Neuhoff and P. C. Shields, "Channel distances and representation," to appear in *Information and Control*.

[2] D. L. Neuhoff and P. C. Shields, "Channel entropy and primitive approximation," *Ann. Prob.* vol. 10, pp. 188-198, 1982.

[3] D. L. Neuhoff and P. C. Shields, "Indecomposable finite state channels and primitive approximation," *IEEE Trans. Inform. Thy.*, vol. IT-28, pp. 11-18, Jan. 1982.

[4] R. Y. Bick, "Continuous alphabet channels: the $\bar{\rho}$-topology, $\bar{\rho}$-finite memory and block coding," Ph.D. Dissertation, Mathematics Department, University of Toledo, June 1982. (Supported by this grant)

3. Spoken papers resulting from the grant

[1] P. C. Shield, "Channels with almost finite memory," Ann. Mtg. of the Amer. Math. Soc., San Antonio, Jan. 1980.

[2] D. L. Neuhoff and P. C. Shields, "Finite state indecomposable channels are almost finite," IEEE International Symposium on Information

Theory, Santa Monica, Feb. 1982.

4. Research personnel

Co-principal investigators

D. L. Neuhoff, Associate Professor of Electrical and Computer Engineering, The University of Michigan.

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Research Assistants

R. Bick, Mathematics Department, University of Toledo. Received the Ph.D. in Mathematics from the University of Toledo, in June 1982. Dissertation title: Continuous Alphabet Channels: The $\bar{\rho}$-Topology, $\bar{\rho}$-Finite Memory and Block Coding.

M. Naraghi-Pour, Computer Information and Control Engineering Program, The University of Michigan. Received M.S.E. in C.I.C.E., December 1981.

K. T. Wu, Computer Information and Control Engineering Program, The University of Michigan.